Optimal Model-Based Control with Limited Communication

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Abstract: Design of optimal controllers for uncertain linear systems subject to communication constraints is investigated in this paper. By following the Model-Based Event-Triggered (MB-ET) control framework we are able to design optimal control laws using the nominal models that are robust to model uncertainties and limited communication. For finite horizon optimal control problems we also provide sub-optimal schedulers that estimate the best time instants to send feedback measurements. The nominal optimal control leftort as well as the cost of communication both in the presence of model uncertainties.

1. INTRODUCTION

The problems of control and estimation under communication constraints have received increased attention in recent years motivated by the extensive use of digital communication networks with limited bandwidth. The communication channel is shared by different applications and in many instances only a reduced number of nodes are able to send information through the network within some specified time interval. This is typical in Networked Control Systems (NCS), where the communication channel may not be available for a given system to communicate at every instant (Moyne and Tilbury [2007]). Also, constraints due to the limited communication and processing capabilities at every node or agent within the network may reduce frequency of transmissions (Tolic and Fierro [2011]). In other applications, it may be desirable not to use the communication channel, even if it is available, due to energy constraints (Araujo [2011], Bernardini and Bemporad [2008]). It becomes essential to determine the conditions under which a dynamical system will remain stable and achieve some desired performance in the presence of model uncertainties, disturbances, and limited feedback information.

An approach for reducing the rate of necessary communication between a system's sensor and controller nodes is the Model-Based Networked Control Systems (MB-NCS) approach. In this framework we use an imperfect model of the physical system or plant to be controlled to generate a control input for the actuator allowing the system to run in open loop for a finite interval of time without need for feedback in this period. The state of the model is then updated when a measurement arrives from the sensor resetting any possible mismatch between plant and model states. It has been shown (Montestruque and Antsaklis [2003]) that using this framework it is possible to stabilize a system by sending periodic measurements to the controller, considerably reducing the number of information packets broadcasted through the network. Different authors have pursued a different, intuitive framework that reduces the rate of communication among agents. In event-triggered broadcasting (Tabuada and Wang [2006], Tabuada [2007], Wang and Lemmon [2008], [2011], Donkers and Heemels [2012]) a subsystem sends its local state to the network only when it is necessary, that is, when a measure of the local subsystem state error is above a specified threshold. These references use a Zero-Order-Hold (ZOH) to compute the control input between events, that is, the updates sent by the sensor node are kept constant at the controller node until new measurements arrive.

The MB-NCS framework with periodic updates by Montestruque and Antsaklis [2003] was extended by Garcia and Antsaklis [2011], [2012], [2013] to consider event-based updates. A similar event-triggered approach using a model of the system was proposed by Lunze and Lehmann [2010]. A common characteristic shared by the control methodologies mentioned above is that, for given system and controller, they are able to reduce network communication for stability. In the present paper we will use the combined Model-Based Event-Triggered (MB-ET) framework (Garcia and Antsaklis [2011]) to maximize the transmission intervals but also consider the control effort. In other words, we address the design of optimal control laws and optimal thresholds for communication in the presence of plant-model mismatch by appropriately weighting the system performance, the control effort, and the communication cost. The approach we follow is to optimize the performance of the nominal system, which can be unstable, and to ensure robust stability for a given class of model uncertainties and for lack of feedback for extended intervals of time.

Similar ideas have been considered by different research groups. For instance, Imer and Basar, [2005], [2006] consider separately the estimation and control problems with limited information when the nominal system is affected by process and measurement noise. In both cases the source node is able to send data through the communication channel only M

times of possible N (M < N). The aim is to find the optimal control law (minimize the average estimation error) by indirectly penalizing channel uses. Molin and Hirche [2009], [2013] also consider the trade-off between control performance and resource utilization, i.e. the cost of updating the controller node using current measurements. The authors study linear systems affected by zero-mean Gaussian noise and they assume that the system parameters and statistics are known, that is, structured model uncertainties are not taken into account in their work. The authors of (Ramesh *et.al.*[2011], Antunes [2013], Cogill, *et.al.*[2007], Xu and Hespanha [2004], MacKunis *et.al.* [2011], Zhang and Hristu-Varsakelis [2005]) also address similar problems.

The contributions of the paper are as follows. We provide a detailed analysis of the robustness of the discretetime Linear Quadratic Regulator (LQR) to parameter uncertainties and absence of feedback measurements for extended periods of time using the MB-ET approach. Similar results are briefly described for the continuous-time case. Then, we consider an approximate solution to the optimal control problem that jointly considers system performance, control effort, and communication costs, all of them in the presence of model uncertainties. The remainder of the paper is organized as follows. In Section 2 we state the discretetime optimal control problem with communication constraints and we also describe an approximation of the solution that considers two sub-problems. In Section 3 we analyze the robustness of one subproblem, the infinite horizon Linear Quadratic Regulator (LQR), to plant model uncertainties and limited feedback measurements. The main results are presented for discrete-time systems but similar results can be obtained for continuous-time systems as well. The design of nominal optimal control inputs and suboptimal events is considered in Section 4. Examples are offered in Section 5. Conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

Consider an uncertain discrete-time linear plant of the form:

$$x(k+1) = Ax(k) + Bu(k)$$
. (1)

The available nominal model of the system is represented by:

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}u(k)$$
 (2)

where $x, \hat{x} \in \mathbb{R}^n$. Define the state error as:

$$e(k) = \hat{x}(k) - x(k)$$
. (3)

Note that the plant can be unstable. It is assumed that transmissions triggered by the occurrence of events do not suffer from delays or losses. The aim is to design an optimal controller for the nominal system (2) that is robust to model uncertainties and to limited feedback information. The latter means that feedback measurements are not always available for control. Therefore, it becomes essential to establish a trade-off between the performance of the control system and the information that can be transmitted. This trade-off can be defined by solving the next optimization problem:

$$\min_{u,\beta} J = x^{T}(N)Q_{N}x(N) + \sum_{k=0}^{N-1} x^{T}(k)Qx(k) + u^{T}(k)Ru(k) + S\beta(k)$$
(4)

where Q and Q_N are real, symmetric, and positive semidefinite matrices, R is a real, symmetric, and positive definite matrix. $\beta(k) \in \{0,1\}$ is a binary decision variable that dictates the communication pattern in the system as follows:

$$\beta(k) = \begin{cases} 1 & measurement \ x_k \ is \ sent \\ 0 & measurement \ x_k \ is \ not \ sent \end{cases}$$
(5)

and S is a positive weighting factor that penalizes network communication. In this paper we follow the Model-Based Event-Triggered (MB-ET) framework (Garcia and Antsaklis [2011], [2013]). This configuration makes use of an explicit model of the plant which is added to the controller node to compute the control input based on the state of the model rather than on the plant state, as represented in Fig. 1.

The goal is to reduce the communication between nodes by reducing the rate at which feedback information is sent to the controller. The MB-ET framework has the advantage that it can provide 'virtual feedback' to control the physical system when no real measurements can be obtained at the controller node due to communication constraints. The combination of the nominal model at the controller and the event-triggering strategy provides a 'virtual feedback' by generating an estimate of the state that is kept close to the real state by maintaining a small state error. In order to obtain the same model state \hat{x} at the sensor node we implement a second identical model at the sensor node that is updated using the measurements x(k) only when $\beta(k) = 1$.

The idea of 'virtual feedback' allows for an approximate solution to (4) which considers the separation of problem (4) into two sub-problems. The first one requires the design of the optimal control for the nominal system for the case when feedback measurements are always available to compute the control input. Since we consider model uncertainties and communication constraints, the optimal controller needs to be robust to both model mismatch and lack of real feedback for intervals of time. Section 3 addresses this problem for the case of infinite horizon optimization problems.



Fig. 1. Model-based event-triggered networked architecture

The second sub-problem aims at minimizing the state error by also considering the cost that needs to be paid every time we decide to send a measurement to update the model and reset the state error. In Section 4 we use the corresponding solution for the first sub-problem for discretetime finite horizon optimization problems along with a Dynamic Programming algorithm that provides the optimal times to send updates.

3. ROBUST LINEAR-QUADRATIC REGULATOR

The robustness of the continuous-time LQR to plant model uncertainties has been analyzed by different authors (Douglas and Athans [1994], Misra [1996], Lin and Olbrot [1996]). In the case of matched uncertainties the LQR guarantees robust stability for any bounded uncertainty of this type. In this section we extend the approach in Lin and Olbrot [1996] for matched uncertainties in order to consider state feedback uncertainties as well. The state uncertainty is characterized by the state error (3) and is the result of limited communication between sensor and controller. We also make use of the MB-ET approach in order to design error events that enable transmission of measurements sensor to controller.

In this section we consider the discrete-time system (1) with model (2) and the state error as defined in (3). We also consider the following assumptions:

a) The nominal system (\hat{A}, \hat{B}) is stabilizable.

b) $B = \hat{B}$

c) We assume matched uncertainty, that is, the uncertainty is in the range of the matrix B. Mathematically we have that there exists a $m \times n$ matrix ϕ such that $\tilde{A} = A - \hat{A} = B\phi$, and ϕ is bounded in the Euclidean sense.

In the absence of model uncertainties we use the nominal system parameters (2) to design a feedback control law u = Kx that minimizes

$$J = \sum_{k=0}^{\infty} x^{T}(k) Fx(k) + x^{T}(k) Qx(k) + u^{T}(k) Ru(k)$$
(6)

where

$$F \ge \phi^T [B^T P B + R]\phi. \tag{7}$$

Q is a real, symmetric, and positive semi-definite matrix, and R is a real, symmetric, and positive definite matrix. The solution of the optimal control problem that minimizes (6) is the discrete-time LQR which provides the feedback gain:

$$K = -[B^T P B + R]^{-1} B^T P \hat{A}$$
(8)

and the matrix P is the solution of the associated discretetime ARE:

$$\hat{A}^{T}P\hat{A} - P + F + Q - \hat{A}^{T}PB[B^{T}PB + R]^{-1}B^{T}P\hat{A} = 0.$$
(9)

Similarly, we can study the robustness of this solution in the presence of both, model uncertainties and absence of feedback at every time instant k, in order to find an event-triggered strategy that provides stability using asynchronous update time intervals.

Theorem 1. System (1) with a model-based input $u = K\hat{x}$ and with model updates based on error events is asymptotically stable for all matched uncertainties satisfying (7) if the updates are triggered when

$$|e| \ge \alpha |x| \tag{10}$$

where $\alpha = \min(\alpha_1, \alpha_2)$, $\alpha_1 = \sigma q / 2c_1$, $\alpha_2 = [\sigma q / 2c_2]^{1/2}$, $0 < \sigma < 1$,

$$\underline{q} = \underline{\sigma}(Q), c_1 = 2 \left| [\hat{A} + B[K + \phi]]^T PBK \right|, and c_2 = \left| K^T B^T PBK \right|.$$

K is the feedback gain given by (8)-(9).

Proof. By using the following relationship:

$$-\hat{A}^{T}PB[B^{T}PB+R]^{-1}B^{T}P\hat{A} =$$

$$\hat{A}^{T}PBK+K^{T}B^{T}P\hat{A}+K[B^{T}PB+R]K$$
(11)

we can rewrite the discrete-time ARE (9) as:

 $[\hat{A} + BK]^T P[\hat{A} + BK] + F + Q + K^T RK - P = 0.$ (12) Let us consider the candidate Lyapunov function $V(x) = x^T Px$ and we evaluate the first forward difference of V(x) along the trajectories of the real system using the model control input $u = K\hat{x}$

$$\begin{split} \Delta V(x) &= V(x(k+1)) - V(x(k)) \\ &= [x^{T}[\hat{A} + BK + \tilde{A}]^{T} + e^{T}K^{T}B^{T}]P[[\hat{A} + BK + \tilde{A}]x + BKe] - x^{T}Px \\ &= x^{T}[\hat{A} + BK + \tilde{A}]^{T}P[\hat{A} + BK + \tilde{A}]x \\ &+ 2x^{T}[\hat{A} + BK + \tilde{A}]^{T}PBKe + e^{T}K^{T}B^{T}PBKe - x^{T}Px \\ &= -x^{T}Qx - x^{T}[F - \phi^{T}[B^{T}PB + R]\phi]x - x^{T}[K + \phi]^{T}R[K + \phi]x \\ &+ 2x^{T}[\hat{A} + BK + \tilde{A}]^{T}PBKe + e^{T}K^{T}B^{T}PBKe \\ &\leq -x^{T}Qx + 2x^{T}[\hat{A} + BK + \tilde{A}]^{T}PBKe + e^{T}K^{T}B^{T}PBKe \\ &\leq -\underline{q}|x|^{2} + 2|[\hat{A} + B[K + \phi]]^{T}PBK||e||x| + |K^{T}B^{T}PBK||e|^{2} \\ &\leq -\underline{q}|x|^{2} + c_{1}|e||x| + c_{2}|e|^{2}. \end{split}$$

Now, by updating the model (the error is set to zero when we update) following the condition in (10), we can bound the error using the term in the right hand side of (10) and we can finally write:

$$V(x(k+1)) - V(x(k)) \le [\sigma - 1]q |x|^2.$$
(13)

Then *V* is guaranteed to decrease for any $\sigma \operatorname{such} 0 < \sigma < 1$ when updating the state of the model using the threshold in Theorem 1. \blacklozenge

Similar results can be obtained for continuous-time systems of the form:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{14}$$

using nominal models expressed by

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}u(t) \tag{15}$$

where $x, \hat{x} \in \mathbb{R}^n$. The state error is defined similar to (3) and we also assume a)-c) above.

In the absence of model uncertainties, we use the nominal system parameters (15) to design a feedback control law u = Kx that minimizes

$$J = \int_{0}^{\infty} \frac{1}{2} [x^{T}(t)Fx(t) + x^{T}(t)Qx(t) + u^{T}(t)Ru(t)]dt \qquad (16)$$

where F is defined as

$$\geq \phi^T R \phi. \tag{17}$$

Q is a real, symmetric, and positive semi-definite matrix, and R is a real, symmetric, and positive definite matrix. The solution of the optimal control problem that minimizes (16) is the LQR which provides the feedback gain:

F

$$K = -R^{-1}B^T P \tag{18}$$

and the matrix *P* is the solution of the associated Algebraic Riccati Equation (ARE) (Douglas and Athans [1994]):

$$\hat{A}^{T}P + P\hat{A} - PBR^{-1}B^{T}P + F + Q = 0.$$
(19)

The robustness of this solution in the presence of both, model uncertainties and absence of continuous feedback is shown next based on event-triggered control updates of the state of the model.

Theorem 2. System (14) with a model-based input $u = K\hat{x}$ and with model updates based on error events is asymptotically stable for all matched uncertainties satisfying (17) if the updates are triggered when

$$\left|e\right| \ge \frac{\sigma q}{\left|K^{T} R K\right|} \left|x\right| \tag{20}$$

where $\underline{q} = \underline{\sigma}(Q)$, $0 < \sigma < 1/2$, *K* is the feedback gain given by (18)-(19).

4. FINITE HORIZON OPTIMAL CONTROL AND OPTIMAL SCHEDULING

Finite horizon optimal control problems are more realistic when considering real problems. The consideration of infinite horizon problems results, in many cases, in simplified controller design steps. This can be easily observed in the continuous and discrete-time LQR above. These simpler controllers can be used in practice, especially, for long or unknown finite horizons. For shorter horizons, we need to formulate the problem differently.

In this section, assuming N is known, we consider our original problem as stated at the beginning of this paper. The discrete-time plant and model are given by (1) and (2) respectively and the cost to be minimized is similar to (4) which includes a penalty for network communication. Additionally we use separate weights F and Q to consider plant model uncertainties. The cost function is given by

$$\min_{u,\beta} J = x^{t} (N)Q_{N}x(N) + \sum_{k=0}^{N-1} x^{T}(k)Fx(k) + x^{T}(k)Qx(k) + u^{T}(k)Ru(k) + S\beta(k).$$
(21)

By following similar ideas as in previous sections we first design the optimal controller for the plant model assuming that feedback measurements are always available. The optimal control input for this case is given by:

$$\iota^*(N-i) = -[B^T P(i-1)B + R]^{-1}B^T P(i-1)\hat{A}x(N-i) \quad (22)$$

where P(i) is recursively computed using:

$$P(i) = [\hat{A} + BK(N-i)]^{T} P(i-1)[\hat{A} + BK(N-i)] + F + Q + K^{T}(N-i)RK(N-i)$$
(23)

Define the Lyapunov function $V(x(k)) = x^T(k)P(k)x(k)$. Using a similar analysis as in Section 3 we can show the following results.

Corollary 3. System (1) with input (22) is stable in the Lyapunov sense for all matched uncertainties satisfying:

$$F \ge \phi^T [B^T P(k)B + R]\phi \tag{24}$$

for all k=0,1,...N.

Note that in order to obtain the optimal control law we need to solve offline the discrete-time LQR, i.e. we need to find K(k) and P(k) before the execution of the system. Then it is possible (knowing a bound on the uncertainty) to check (24) in advance. When (24) holds we know the system is stable and the optimal cost of the form (6) with no

communication penalty is finite when measurements are available at every time k. Then we can proceed to select an appropriate weight on the communication to restrict measurement updates from the sensor to the controller.

We are now in the position to approach the second problem that was introduced in the beginning of this paper in a more formal way. In Section 3 we were able to reduce the communication rate between sensor and controller while using the optimal control law and the estimates generated by the model. However, the communication pattern was not optimal. Next we use the error nominal dynamics and the selected communication factor S in order to design suboptimal update events. The difficulty in obtaining optimal control laws and optimal schedulers was highlighted by different authors Molin and Hirche [2009], [2013], Ramesh et.al. [2011], and Xu and Hespanha [2004]. The problem discussed in this paper is more challenging by considering plan-model mismatch. In the following we discuss an approximate solution to our problem. This approximation to solve the optimal scheduling problem can be seen as the minimization of the deviation of the system performance from the nominal closed-loop performance by considering the cost that needs to be paid because of the updating of the state of the model and the resetting of the state error.

The error dynamics are given by:

$$e(k+1) = \hat{x}(k+1) - x(k+1)$$

$$= \hat{A}\hat{x}(k) + Bu(k) - Ax(k) - Bu(k)$$

$$= \hat{A}e(k) - \tilde{A}x(k).$$
(25)

Since the uncertainty \tilde{A} is not known, we use the nominal error dynamics, i.e.

$$e(k+1) = \hat{A}e(k)$$
. (26)

Furthermore, when the sensor decides to send a measurement update, which makes $\beta(k) = 1$, we reset the error to zero. Then the complete nominal error dynamics can be represented by the following equation:

$$e(k+1) = \hat{A}e(k) - \beta(k)\hat{A}e(k) = [1 - \beta(k)]\hat{A}e(k).$$
(27)

It is clear that, in the nominal case, when no model uncertainties exist, once we update the model the state error is equal to zero for the remaining time instants. However, in a real problem the state error dynamics are disturbed by the state of the real system which is propagated by means of the model uncertainties as expressed in (25). Then, using the available model dynamics we implement the nominal optimal control input and the sub-optimal scheduler that results from the following optimization problem:

$$\min_{\beta} J_e = e^T(N)Q_N e(N) + \sum_{k=0}^{N-1} e^T(k)Qe(k) + S\beta(k) \quad (28)$$

subject to $e(k+1) = [1-\beta(k)]\hat{A}e(k)$
 $\beta(k) \in \{0,1\}.$

In order to solve problem (28) we use Dynamic Programming in the form of look up tables. The main reason for using Dynamic Programming is that, although the error will be finely quantized, the decision variable $\beta(k)$ only takes two possible values, which reduces the amount of computations performed by the Dynamic Programming algorithm. The sensor operations at time k are reduced to measure the real state, compute and quantize the state error and determine if the current measurement has to be transmitted by looking at the corresponding table entries which are computed offline. The table size depends only on the horizon N and the error quantization levels.

5. EXAMPLE

Example 1. Consider the nominal model of an unstable discrete-time second order system given by:



Fig. 2. Response of the real uncertain system with limited feedback (top). States of the model used to control the system (bottom)



Fig. 3. State error used to determine the instants at which a feedback measurement has to be transmitted (top). Communication pattern $\beta(k)$ (bottom)



Fig. 4. Response of the nominal system receiving feedback measurements at every time k

Let the unknown dynamics of the unstable real system used in the next simulation example be:

$$A = \begin{bmatrix} 1.25 & 0.48 \\ -0.73 & -0.81 \end{bmatrix}$$

The model-based controlled system is intended to operate over a finite period of N=30 stages. The parameters in the optimization problem are as follows: Q = R = I, S = 1, $Q_N = 2I$. The unknown initial conditions of the system are given by $x(0) = [1.9 - 1.4]^T$. Since these initial conditions are unknown to the controller, the model is initialized using $\hat{x}(0) = [0 \ 0]^T$. The computation of the Control gains at every time instant and the solution of the Dynamic Programming optimization problem are computed offline and stored accordingly in the controller and sensor nodes. The results of the simulation are shown in Fig. 2 - Fig. 4 using linear interpolation to connect the consecutive values in the response of the system and model.

Fig. 2 shows both the states of the real system and the states of the model that are used in the computation of the control input when feedback measurements are unavailable. The top portion of Fig. 3 shows the state error. For the different combinations of the state error we find the corresponding entry on the table that contains the solution of the Dynamic Programming optimization problem. At every time instant the sensor decides to send or not the measurement of the state of the system based on the values of the error, those decisions are represented at the bottom of Fig. 3. An important difference with respect to the error threshold designed in Section 3 is that the solution of the problem in (28) considers the dynamics (nominal) of the error in the design of the transmission events. By including a prediction of the behavior of the state error we are also able to predict the consequences, as measured by the computation of the optimal cost, of updating or not the model at a given time instant k. We can see, for instance, that some combinations of state error values will result in different transmission decisions depending at which of the N stages we are operating at that moment.

It is also important to note that in the absence of model uncertainties we obtain the response of the nominal system as if measurements were always available. Fig. 4 shows the response of the nominal system when feedback measurements are available at every time instant k. The behavior of the overall controlled system which consists on the MB-ET framework with optimal control input and optimal scheduler is comparable to that of the nominal system for which the optimal control law is designed, and the difference is considerably reduced as the uncertainties diminish.

6. CONCLUSIONS

The design of optimal controllers for unknown systems represents a challenging task. An approach that has been studied in the literature is to design an optimal controller for a nominal system and analyze its robustness to a range of uncertainties. In this paper we considered a more complex version of this problem in which feedback information is also limited. In addition to designing an optimal control input, we also addressed the problem of finding the optimal instants at which we should transmit feedback measurements. Our approach, which consists of model-based estimates of the state and of error events, provides a practical and promising methodology that considers the overall performance of the communication constrained system.

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